Quantum neural networks

Kerstin Beer

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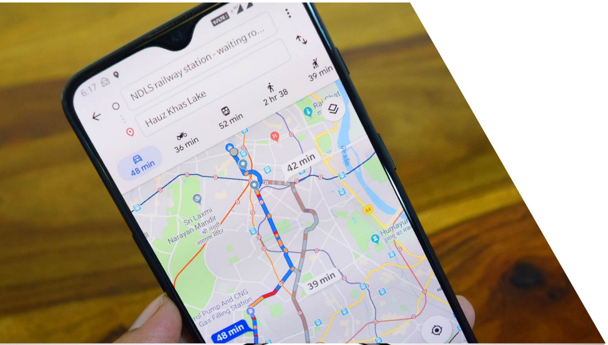

NDLS railway station - waiting ro...
Hauz Khas Lake
48 min
36 min
52 min
2 hr 38
39 min
42 min
Sri Laxmi
Narayan Mandir
39 min
48 min

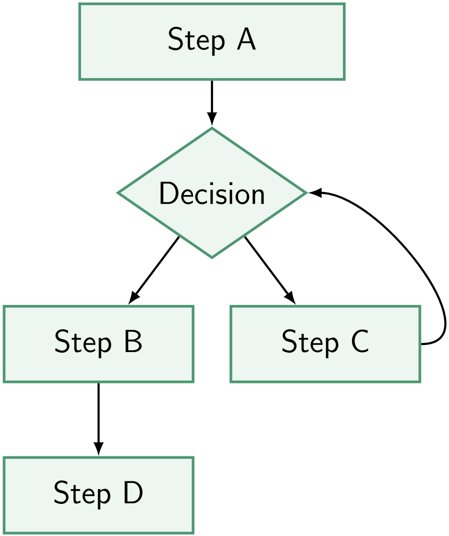

Google

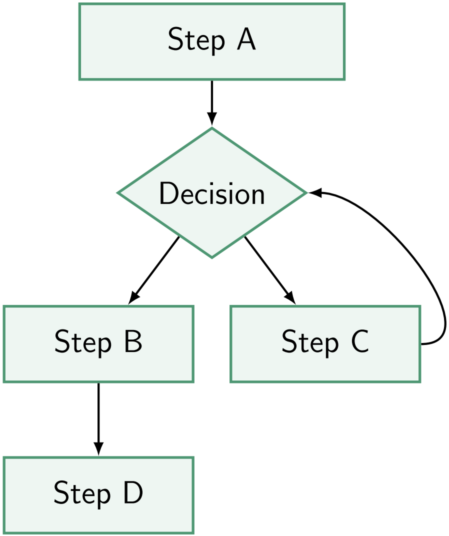

NDLS railway station - waiting ro...
Hauz Khas Lake
48 min
36 min
52 min
2 hr 38
39 min
42 min
Sri Laxmi Narayan Mandir
39 min
48 min
Google

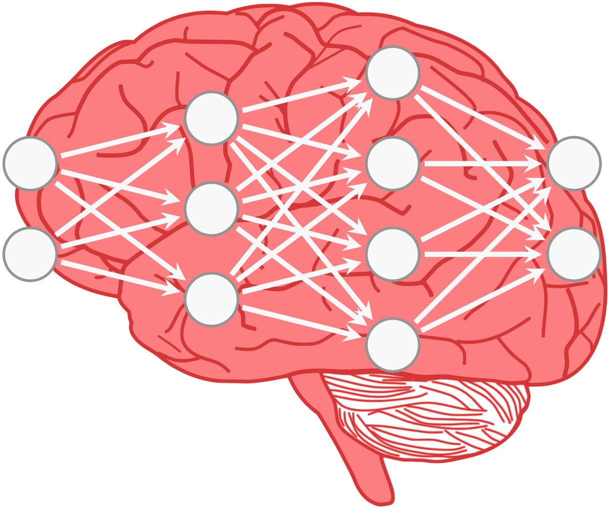

NDLS railway station - waiting ro...
Hauz Khas Lake
48 min
36 min
52 min
2 hr 38
39 min
42 min
39 min
Sri Laxmi Narayan Mandir
48 min
Google

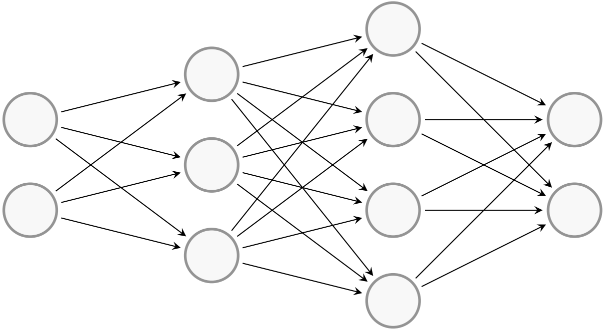
NDLS railway station - waiting ro...
Hauz Khas Lake
48 min
36 min
52 min
2 hr 38
39 min
42 min
Sri Laxmi Narayan Mandir
39 min
Petrol Pump And CNG Gas Filling Station
48 min
Google

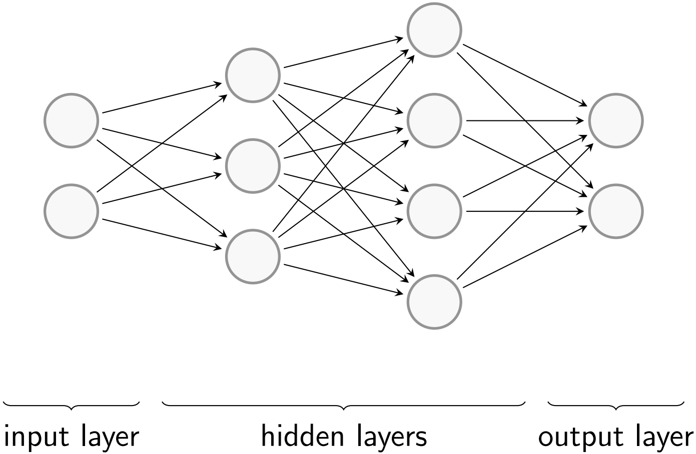

NDLS railway station - waiting ro...
Hauz Khas Lake
48 min
36 min
52 min
2 hr 38
39 min
42 min
Sri Laxmi
Narayan Mandir
39 min
48 min
Google


Step A
Decision
Step B
Step C
Step D


Step A
Decision
Step B
Step C
Step D



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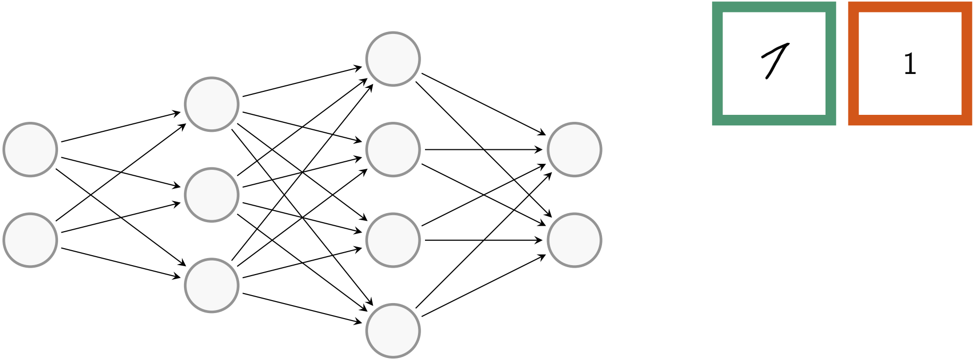
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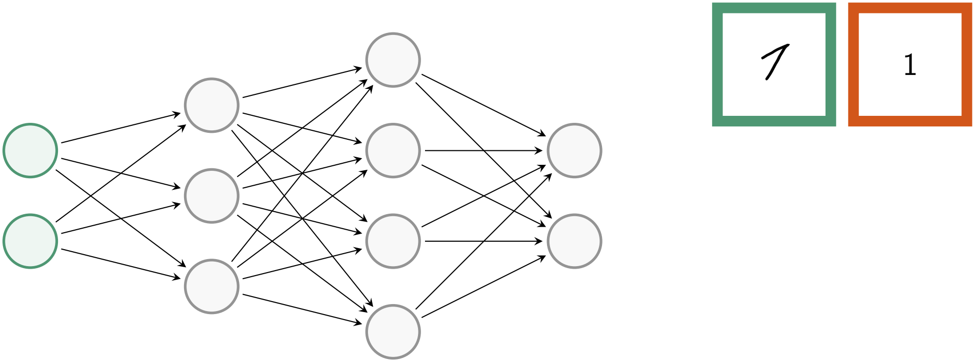
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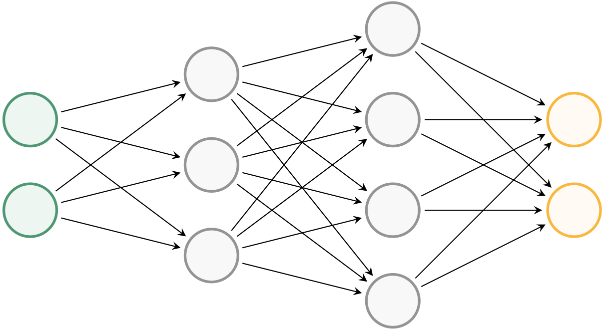

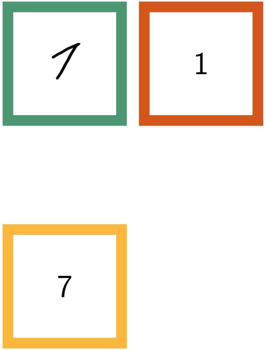
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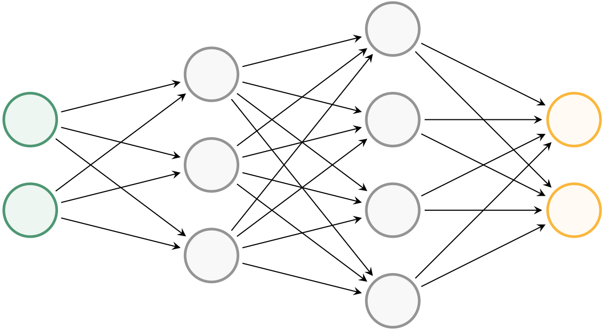

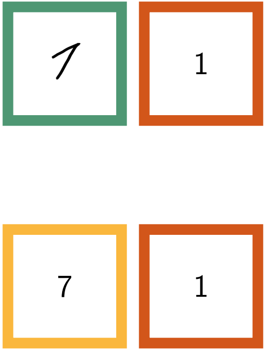
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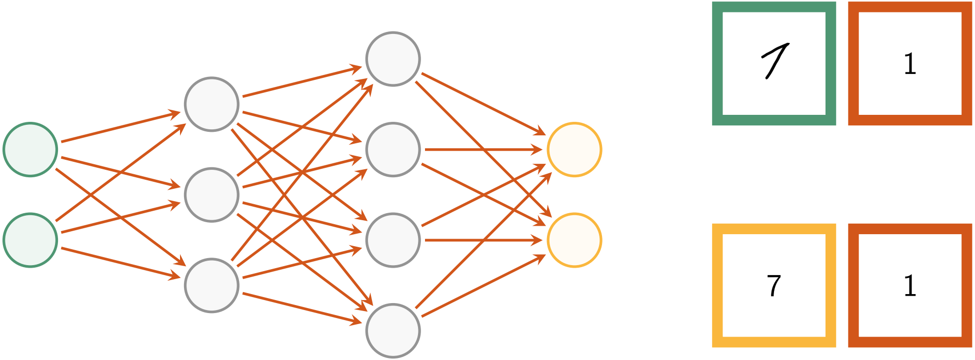
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7
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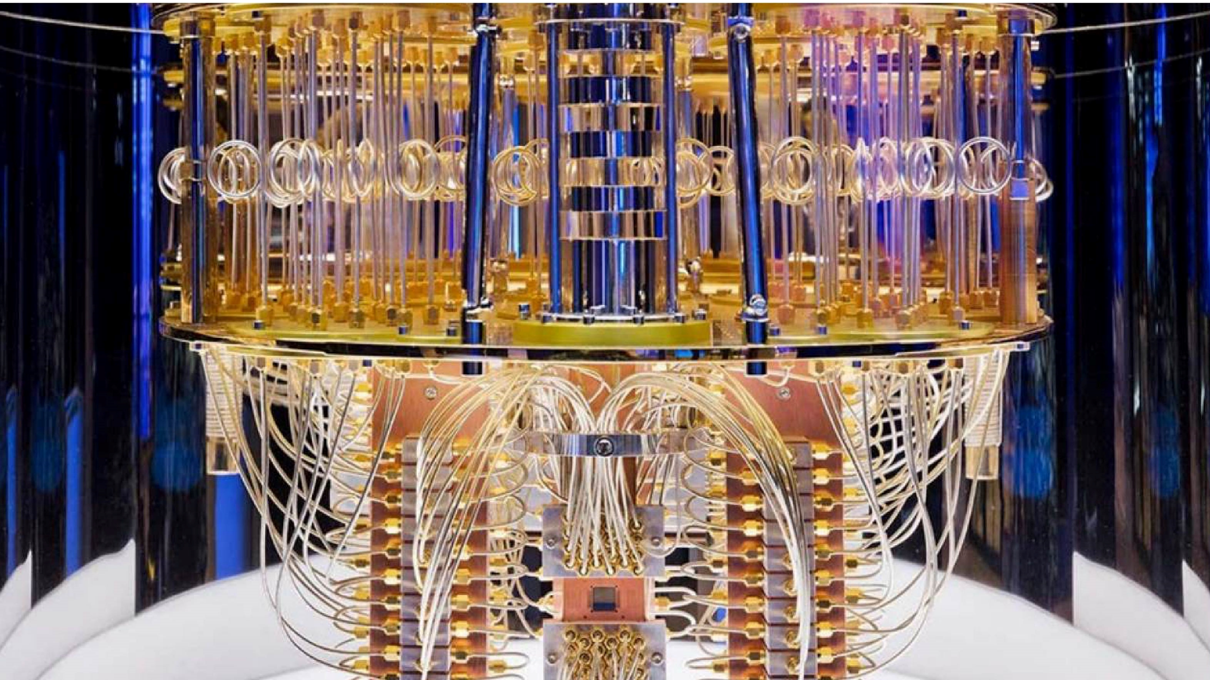
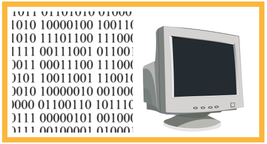
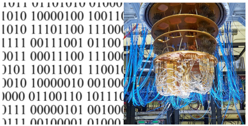
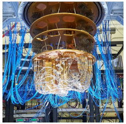


		algorithm	
		classical	quantum
data	classical		
	quantum	$\vert Q\rangle$ 	$\vert Q\rangle$ 

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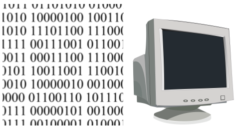
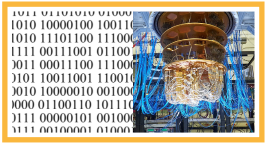
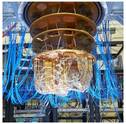
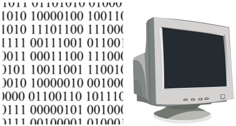
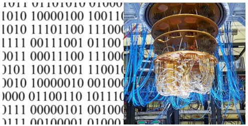
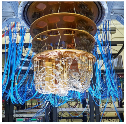
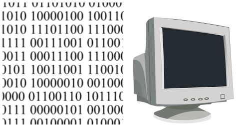
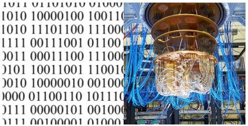
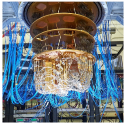
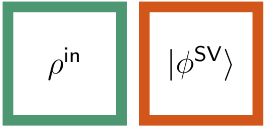
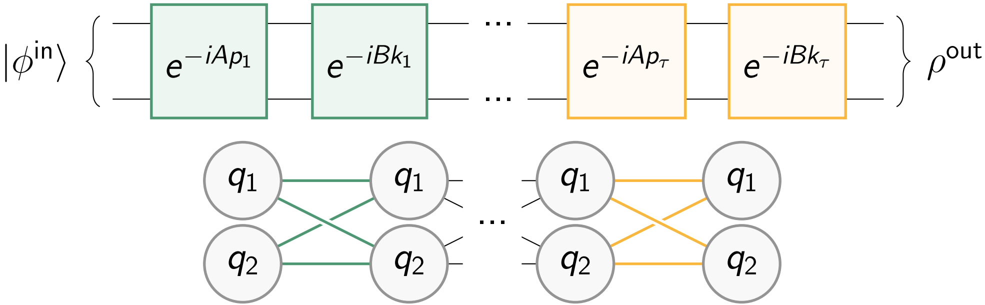
		classical	quantum
data	classical		
	quantum	$\lvert Q \rangle$ 	$\lvert Q \rangle$ 

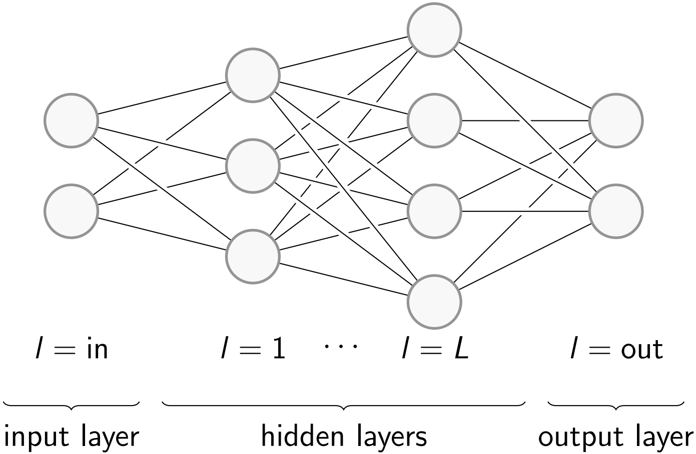
		algorithm	
		classical	quantum
data	classical		
	quantum	$\lvert Q\rangle$ 	$\lvert Q\rangle$ 

		algorithm	
		classical	quantum
data	classical		
	quantum	$\vert Q\rangle$ 	$\vert Q\rangle$ 


ρ^{in}
$|\phi^{\mathsf{SV}}\rangle$



$|\phi^{\text{in}}\rangle$
e^{-iAp_1}
e^{-iBk_1}
e^{-iAp_τ}
e^{-iBk_τ}
ρ^{out}
q_1
q_2
q_1
q_2
q_1
q_2
q_1
q_2



$l = \text{in}$
$l = 1$
$\cdots$
$l = L$
$l = \text{out}$
input layer
hidden layers
output layer

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ARTICLE

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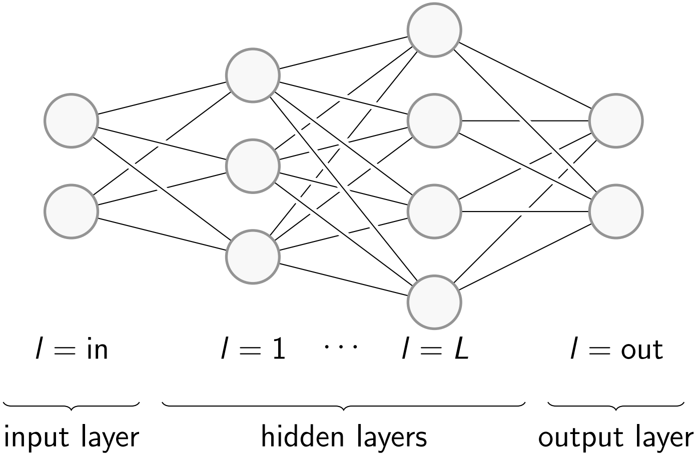
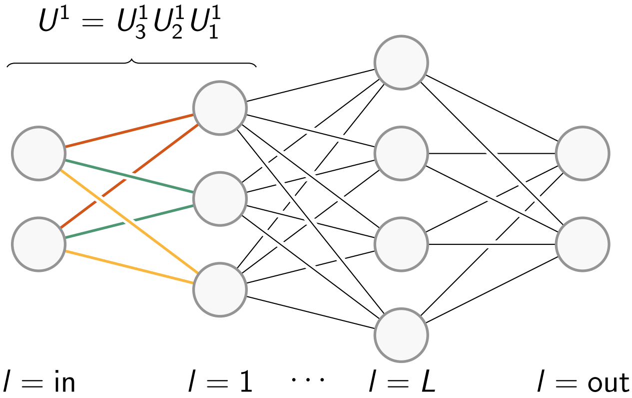
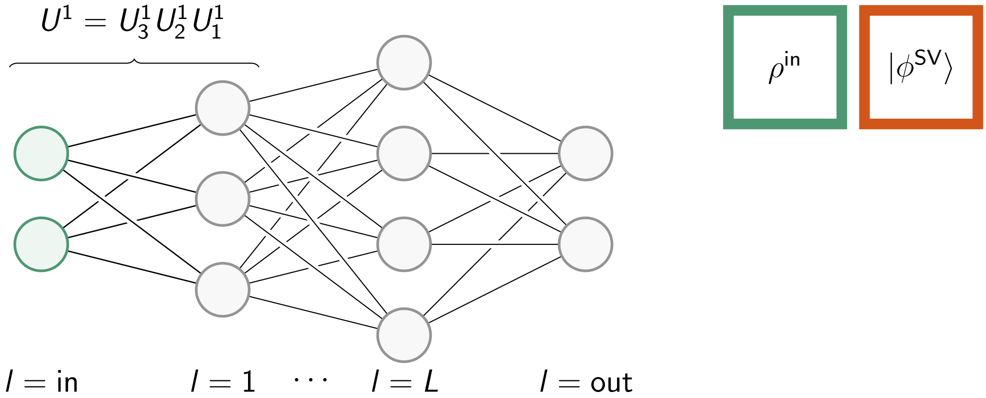
Training deep quantum neural networks

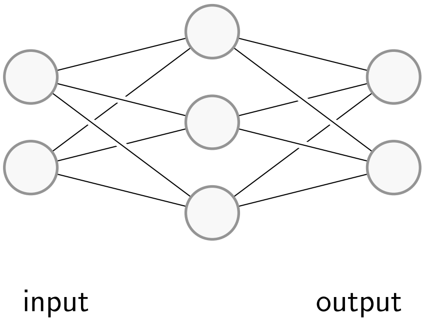
Kerstin Beer[1*], Dmytro Bondarenko[1], Terry Farrelly[1,2], Tobias J. Osborne[1], Robert Salzmann[1,3], Daniel Scheiermann[1] & Ramona Wolf[1]

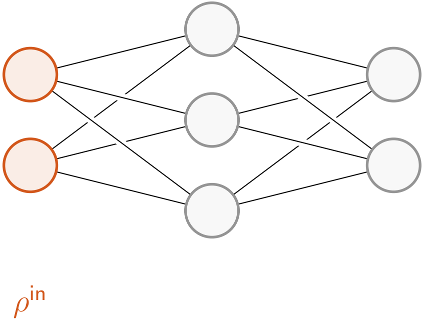
Neural networks enjoy widespread success in both research and industry and, with the advent of quantum technology, it is a crucial challenge to design quantum neural networks for fully quantum learning tasks. Here we propose a truly quantum analogue of classical neurons, which form quantum feedforward neural networks capable of universal quantum computation. We describe the efficient training of these networks using the fidelity as a cost function, providing both classical and efficient quantum implementations. Our method allows for fast optimisation with reduced memory requirements: the number of qudits required scales with

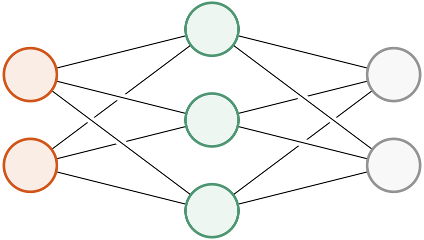

$l = \text{in}$
$l = 1$ $\cdots$ $l = L$
$l = \text{out}$
input layer
hidden layers
output layer


$U^1 = U^1_3 U^1_2 U^1_1$
$l = \text{in}$
$l = 1$
$\cdots$
$l = L$
$l = \text{out}$

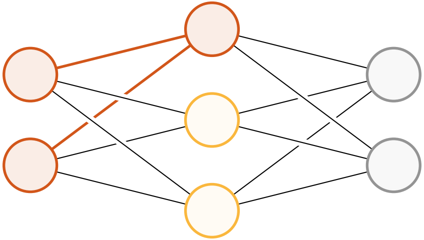

$U^1 = U_3^1 U_2^1 U_1^1$
ρ^{in}
$|\phi^{\text{SV}}\rangle$
$l = \text{in}$
$l = 1$
$\cdots$
$l = L$
$l = \text{out}$


input
output

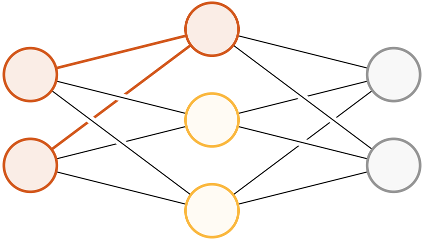

ρ^{in}



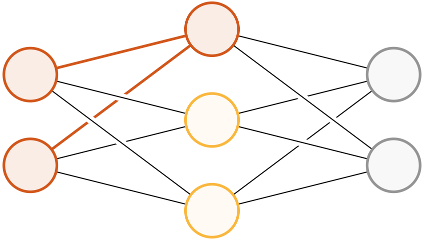
$$\rho^{\mathsf{in}} \otimes |000\rangle\langle 000|$$



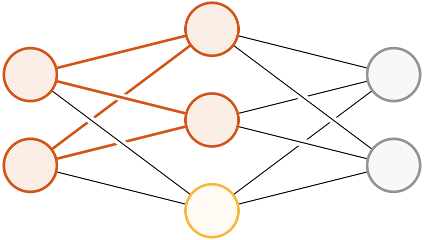
$$U_1^1(\rho^{\text{in}} \otimes |000\rangle\langle 000|)U_1^{1\dagger}$$



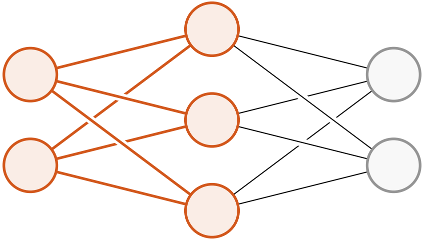
$$U_1^1 \otimes \mathbb{1} \otimes \mathbb{1} (\rho^{\text{in}} \otimes |000\rangle \langle 000|) {U_1^1}^\dagger \otimes \mathbb{1} \otimes \mathbb{1}$$



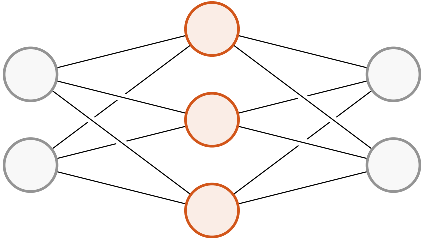
$$U_1^1 (\rho^{\text{in}} \otimes |000\rangle \langle 000|) U_1^{1\dagger}$$



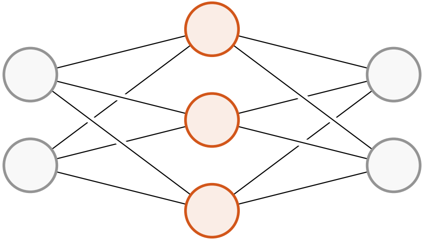
$$U_2^1 U_1^1 (\rho^{\mathrm{in}} \otimes |000\rangle \langle 000|) U_1^{1\dagger} U_2^{1\dagger}$$



$$\color{orange}{U_3^1}\, U_2^1\, U_1^1 (\rho^{\text{in}} \otimes |000\rangle\langle 000|)\, U_1^{1\dagger}\, U_2^{1\dagger}\, \color{orange}{U_3^{1\dagger}}$$

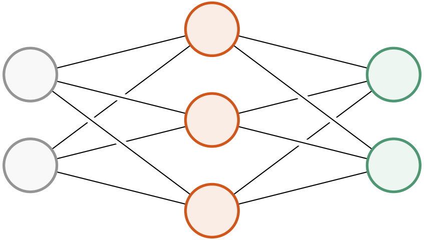


$$\mathrm{tr}_{\mathrm{in}}\left(U_3^1 U_2^1 U_1^1 (\rho^{\mathrm{in}} \otimes |000\rangle \langle 000|) {U_1^1}^\dagger {U_2^1}^\dagger {U_3^1}^\dagger \right)$$

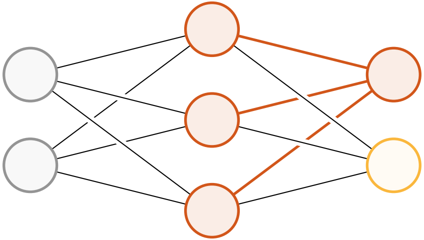


$$\mathrm{tr}_{\mathrm{in}}\left(U_3^1 U_2^1 U_1^1 (\rho^{\mathrm{in}} \otimes |000\rangle\langle 000|) {U_1^1}^\dagger {U_2^1}^\dagger {U_3^1}^\dagger\right)$$

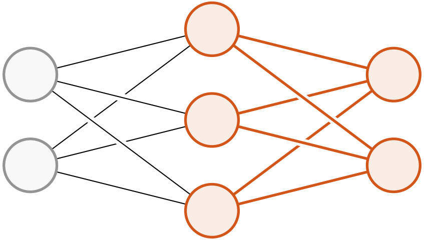
Dissipative quantum neural network



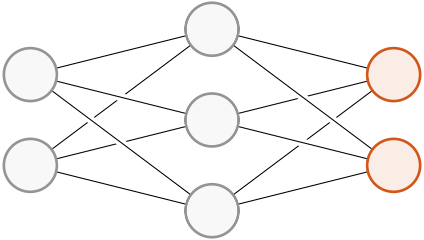
$$\mathrm{tr_{in}}\left(U_3^1 U_2^1 U_1^1 (\rho^{\mathrm{in}} \otimes |000\rangle\langle 000|) {U_1^1}^\dagger {U_2^1}^\dagger {U_3^1}^\dagger\right) \otimes |00\rangle\langle 00|$$



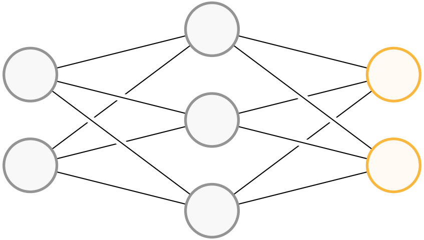
$$U_1^{\text{out}}(\text{tr}_{\text{in}}(U_3^1 U_2^1 U_1^1(\rho^{\text{in}} \otimes |000\rangle\langle 000|) U_1^{1\dagger} U_2^{1\dagger} U_3^{1\dagger}) \otimes |00\rangle\langle 00|) U_1^{\text{out}\dagger}$$



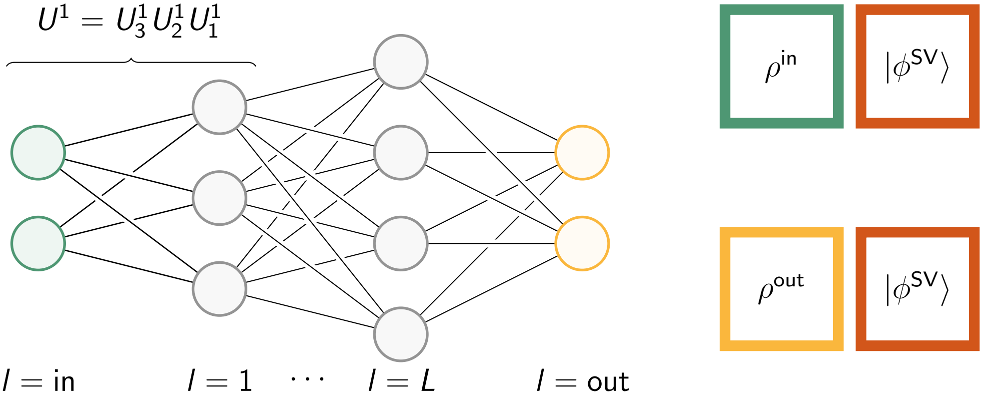
$$U_2^{\text{out}} U_1^{\text{out}} \left(\text{tr}_{\text{in}} \left(U_3^1 U_2^1 U_1^1 \left(\rho^{\text{in}} \otimes |000\rangle \langle 000| \right) {U_1^1}^\dagger {U_2^1}^\dagger {U_3^1}^\dagger \right) \otimes |00\rangle \langle 00| \right) {U_2^{\text{out}}}^\dagger {U_1^{\text{out}}}^\dagger$$



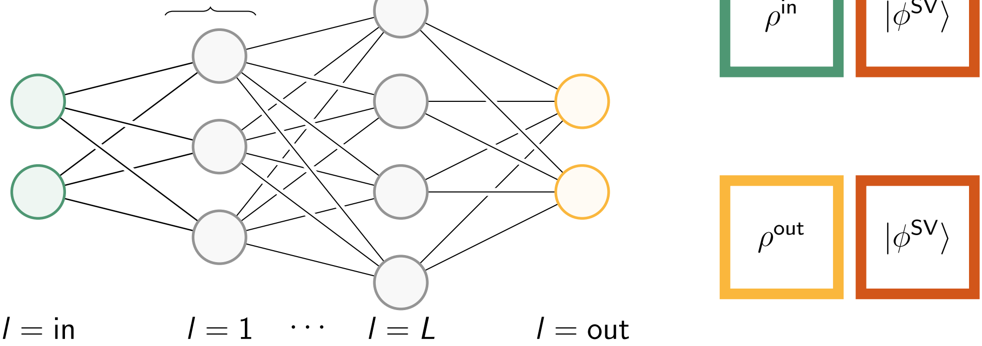
$$\mathrm{tr}_{\mathrm{hidden}}\left(U_2^{\mathrm{out}} U_1^{\mathrm{out}} \left(\mathrm{tr}_{\mathrm{in}}\left(U^1 \left(\rho^{\mathrm{in}} \otimes |000\rangle \langle 000|\right) U^{1\dagger}\right) \otimes |00\rangle \langle 00|\right) U_1^{\mathrm{out}\dagger} U_2^{\mathrm{out}\dagger}\right)$$

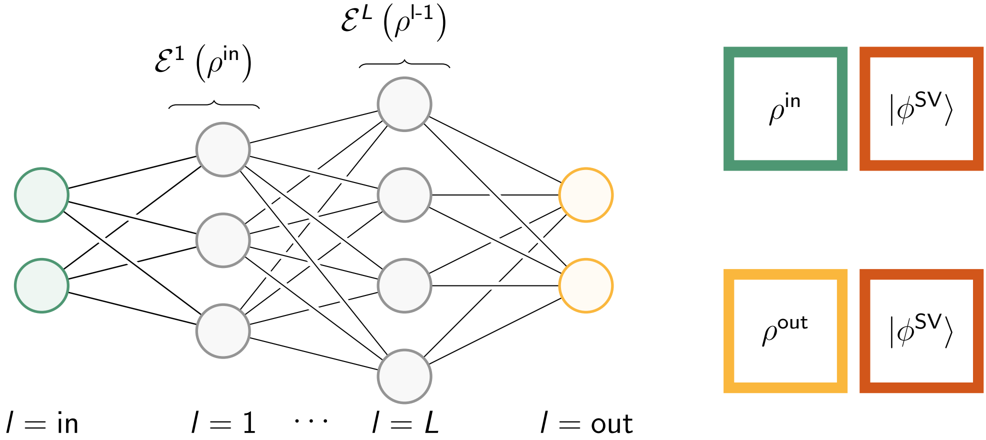


ρ^{out}


$U^1 = U_3^1 U_2^1 U_1^1$
$l = \text{in}$
$l = 1$
$\cdots$
$l = L$
$l = \text{out}$
ρ^{in}
$|\phi^{\text{SV}}\rangle$
ρ^{out}
$|\phi^{\text{SV}}\rangle$

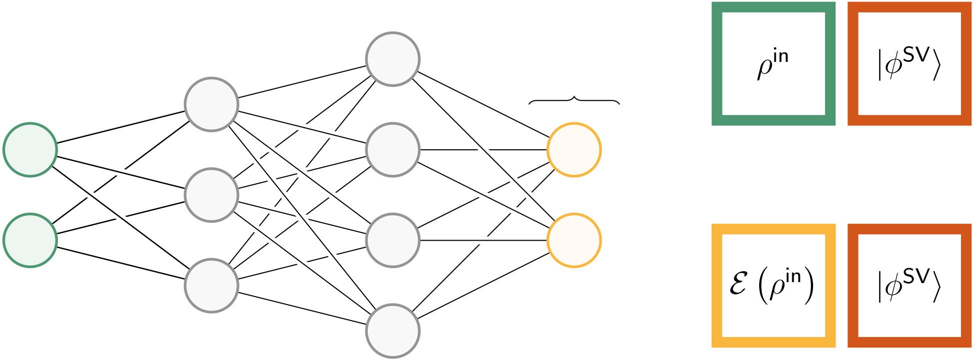
$$\mathrm{tr}_{\mathrm{in}}\left(U_3^1 U_2^1 U_1^1 (\rho^{\mathrm{in}} \otimes |000\rangle\langle 000|) U_1^{1\dagger} U_2^{1\dagger} U_3^{1\dagger}\right) = \mathcal{E}^1\left(\rho^{\mathrm{in}}\right)$$





$\mathcal{E}^1\left(\rho^{\text{in}}\right)$
$\mathcal{E}^L\left(\rho^{\text{l-1}}\right)$
$l = \text{in}$
$l = 1$
$\cdots$
$l = L$
$l = \text{out}$
ρ^{in}
$|\phi^{\text{SV}}\rangle$
ρ^{out}
$|\phi^{\text{SV}}\rangle$

$$\rho^{\mathrm{out}} = \mathcal{E}^{\mathrm{out}}\left(\mathcal{E}^{L}\left(\ldots\mathcal{E}^{2}\left(\mathcal{E}^{1}\left(\rho^{\mathrm{in}}\right)\right)\ldots\right)\right) = \mathcal{E}\left(\rho^{\mathrm{in}}\right)$$



compare

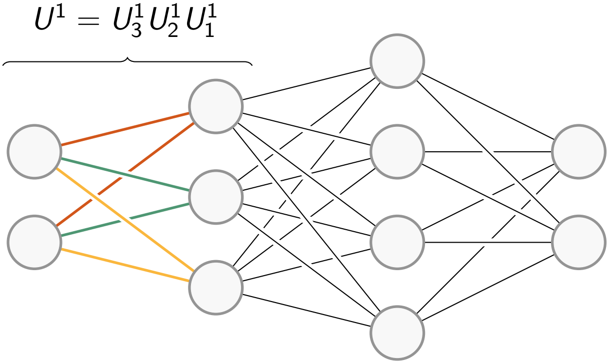
$\mathcal{E}\left(\rho^{\text{in}}\right)$ $|\phi^{\text{SV}}\rangle$

compare

$\mathcal{E}\left(\rho^{\mathrm{in}}\right)$ $|\phi^{\mathrm{SV}}\rangle$

with

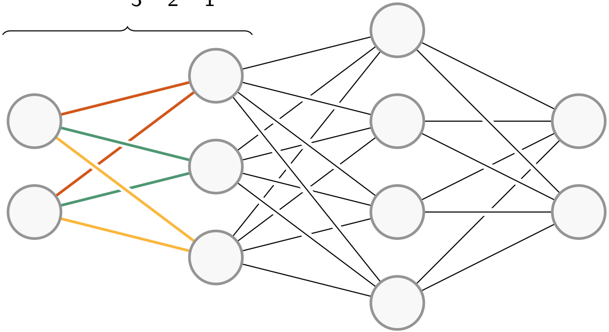
$$\mathcal{L}_{\mathrm{SV}} \equiv \frac{1}{S}\sum_{x=1}^{S} \langle\phi_x^{\mathrm{SV}}|\,\mathcal{E}\left(\rho_x^{\mathrm{in}}\right)|\phi_x^{\mathrm{SV}}\rangle$$


$U^1 = U_3^1 U_2^1 U_1^1$

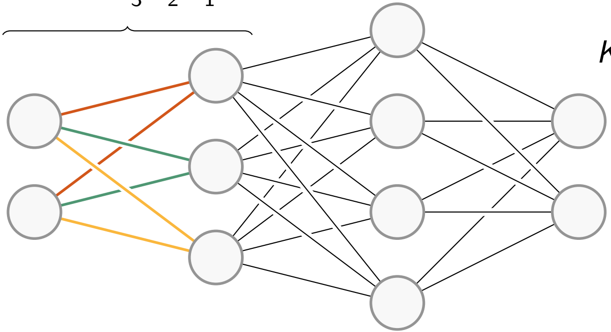
update

$$U_j^l(t+\epsilon) = e^{i\epsilon K_j^l(t)} U_j^l(t)$$

$$U^1 = U_3^1 U_2^1 U_1^1$$



$$U^1 = U^1_3 U^1_2 U^1_1$$



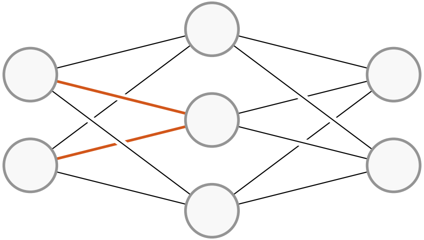
update

$$U^l_j(t+\epsilon) = e^{i\epsilon K^l_j(t)} U^l_j(t)$$

$$K^l_j(t) = \frac{\eta 2^{m_l - 1} i}{S} \sum_x \mathrm{tr}_{\mathrm{rest}} \left\{ M^l_j(t) \right\}$$

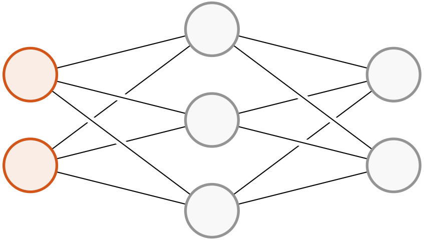
$$K_2^1 = \frac{2^2\eta}{S} \sum_{x=1} \mathrm{tr}_{\mathrm{rest}} \Big[U_2^1 U_1^1 \left(\rho_x^{\mathrm{in}} \otimes |000\rangle \langle 000| \right) U_1^{1\dagger} U_2^{1\dagger},$$

$$U_3^{1\dagger} \left(\mathbb{1}_{1,2} \otimes \mathrm{tr}_{\mathrm{out}} \left(U_1^{3\dagger} U_2^{3\dagger} \left(\mathbb{1}_{3,4,5} \otimes |\phi_x^{\mathrm{SV}}\rangle \langle \phi_x^{\mathrm{SV}}| \right) U_2^3 U_1^3\right)\right) U_3^1 \Big]$$



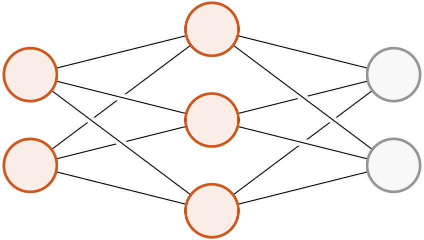
$$K_2^1 = \frac{2^2\eta}{S} \sum_{x=1} \mathrm{tr}_{\mathrm{rest}} \Big[U_2^1 U_1^1 \left(\rho_x^{\mathrm{in}} \otimes |000\rangle\langle 000| \right) U_1^{1\dagger} U_2^{1\dagger},$$

$$U_3^{1\dagger} \left(\mathbb{1}_{1,2} \otimes \mathrm{tr}_{\mathrm{out}} \left(U_1^{3\dagger} U_2^{3\dagger} \left(\mathbb{1}_{3,4,5} \otimes |\phi_x^{\mathrm{SV}}\rangle\langle \phi_x^{\mathrm{SV}}| \right) U_2^3 U_1^3 \right) \right) U_3^1 \Big]$$

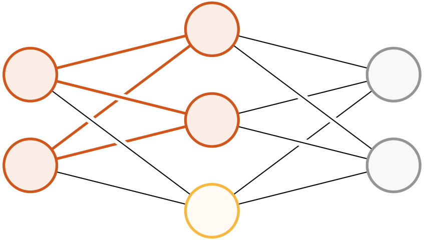


$$K_2^1 = \frac{2^2\eta}{S} \sum_{x=1} \mathrm{tr}_{\mathrm{rest}} \Big[U_2^1 U_1^1 \left(\rho_x^{\mathrm{in}} \otimes |000\rangle \langle 000| \right) U_1^{1\,\dagger} U_2^{1\,\dagger},$$

$$U_3^{1\,\dagger} \left(\mathbb{1}_{1,2} \otimes \mathrm{tr}_{\mathrm{out}} \left(U_1^{3\,\dagger} U_2^{3\,\dagger} \left(\mathbb{1}_{3,4,5} \otimes |\phi_x^{\mathrm{SV}}\rangle \langle \phi_x^{\mathrm{SV}}| \right) U_2^3 U_1^3 \right) \right) U_3^1 \Big]$$

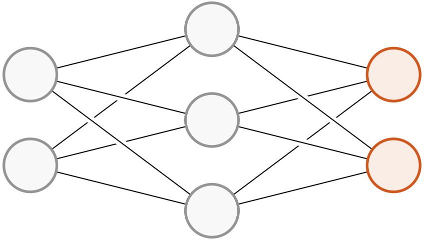


$$K_2^1 = \frac{2^2 \eta}{S} \sum_{x=1} \mathrm{tr}_{\mathrm{rest}} \Big[U_2^1 U_1^1 \left(\rho_x^{\mathrm{in}} \otimes |000\rangle \langle 000| \right) U_1^{1\dagger} U_2^{1\dagger},$$
$$U_3^{1\dagger} \left(\mathbb{1}_{1,2} \otimes \mathrm{tr}_{\mathrm{out}} \left(U_1^{3\dagger} U_2^{3\dagger} \left(\mathbb{1}_{3,4,5} \otimes |\phi_x^{\mathrm{SV}}\rangle \langle \phi_x^{\mathrm{SV}}| \right) U_2^3 U_1^3 \right) \right) U_3^1 \Big]$$



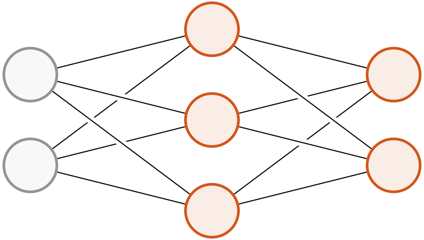
$$K_2^1 = \frac{2^2\eta}{S}\sum_{x=1} \mathrm{tr}_{\mathrm{rest}}\Big[U_2^1 U_1^1 \left(\rho_x^{\mathrm{in}} \otimes |000\rangle\langle 000|\right) U_1^{1\dagger} U_2^{1\dagger},$$

$$U_3^{1\dagger}\left(\mathbb{1}_{1,2} \otimes \mathrm{tr}_{\mathrm{out}}\left(U_1^{3\dagger} U_2^{3\dagger}\left(\mathbb{1}_{3,4,5} \otimes |\phi_x^{\mathrm{SV}}\rangle\langle\phi_x^{\mathrm{SV}}|\right) U_2^3 U_1^3\right)\right) U_3^1\Big]$$



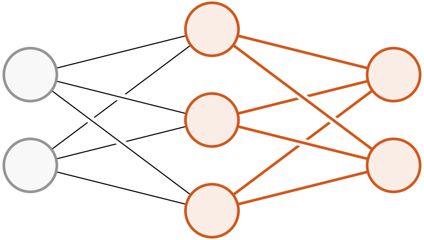
$$K_2^1 = \frac{2^2\eta}{S} \sum_{x=1} \mathrm{tr}_{\mathrm{rest}} \Big[U_2^1 U_1^1 \, \big(\rho_x^{\mathrm{in}} \otimes |000\rangle\langle 000|\big) \, U_1^{1\dagger} U_2^{1\dagger},$$

$$U_3^{1\dagger} \big(\mathbb{1}_{1,2} \otimes \mathrm{tr}_{\mathrm{out}} \big(U_1^{3\dagger} U_2^{3\dagger} \big(\mathbb{1}_{3,4,5} \otimes |\phi_x^{\mathrm{SV}}\rangle\langle\phi_x^{\mathrm{SV}}|\big) U_2^3 U_1^3\big)\big) U_3^1 \Big]$$



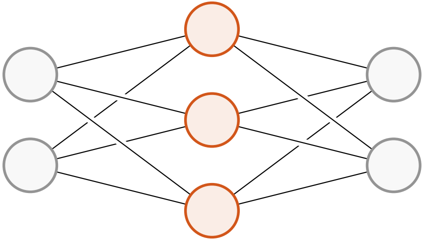
$$K_2^1 = \frac{2^2\eta}{S} \sum_{x=1} \mathrm{tr}_{\mathrm{rest}} \Big[U_2^1 U_1^1 \left(\rho_x^{\mathrm{in}} \otimes |000\rangle\langle 000|\right) U_1^{1\dagger} U_2^{1\dagger},$$

$$U_3^{1\dagger} \left(\mathbb{1}_{1,2} \otimes \mathrm{tr}_{\mathrm{out}} \left(U_1^{3\dagger} U_2^{3\dagger} \left(\mathbb{1}_{3,4,5} \otimes |\phi_x^{\mathrm{SV}}\rangle\langle\phi_x^{\mathrm{SV}}|\right) U_2^3 U_1^3\right)\right) U_3^1 \Big]$$



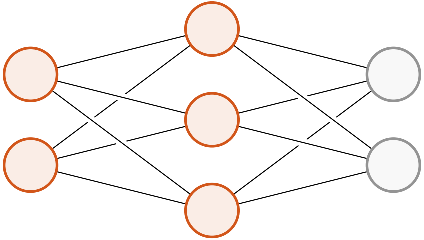
$$K_2^1 = \frac{2^2\eta}{S}\sum_{x=1} \mathrm{tr}_{\mathrm{rest}}\Big[U_2^1 U_1^1 \left(\rho_x^{\mathrm{in}} \otimes |000\rangle\langle 000|\right) U_1^{1\dagger} U_2^{1\dagger},$$

$$U_3^{1\dagger}\left(\mathbb{1}_{1,2} \otimes \mathrm{tr}_{\mathrm{out}}\left(U_1^{3\dagger} U_2^{3\dagger}\left(\mathbb{1}_{3,4,5} \otimes |\phi_x^{\mathrm{SV}}\rangle\langle\phi_x^{\mathrm{SV}}|\right) U_2^3 U_1^3\right)\right) U_3^1\Big]$$



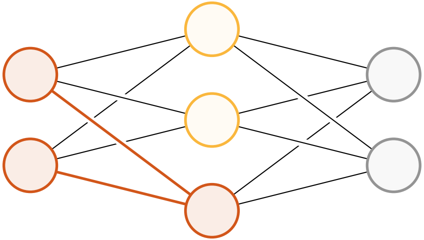
$$K_2^1 = \frac{2^2\eta}{S} \sum_{x=1} \mathrm{tr}_{\mathrm{rest}} \Big[U_2^1 U_1^1 \left(\rho_x^{\mathrm{in}} \otimes |000\rangle \langle 000| \right) U_1^{1\dagger} U_2^{1\dagger},$$

$$U_3^{1\dagger} \left(\mathbb{1}_{1,2} \otimes \mathrm{tr}_{\mathrm{out}} \left(U_1^{3\dagger} U_2^{3\dagger} \left(\mathbb{1}_{3,4,5} \otimes |\phi_x^{\mathrm{SV}}\rangle \langle \phi_x^{\mathrm{SV}}| \right) U_2^3 U_1^3 \right) \right) U_3^1 \Big]$$



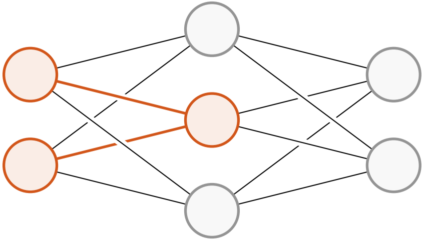
$$K_2^1 = \frac{2^2\eta}{S}\sum_{x=1} \mathrm{tr}_{\mathrm{rest}}\Big[U_2^1 U_1^1 \left(\rho_x^{\mathrm{in}} \otimes |000\rangle\langle 000|\right) U_1^{1\dagger} U_2^{1\dagger},$$

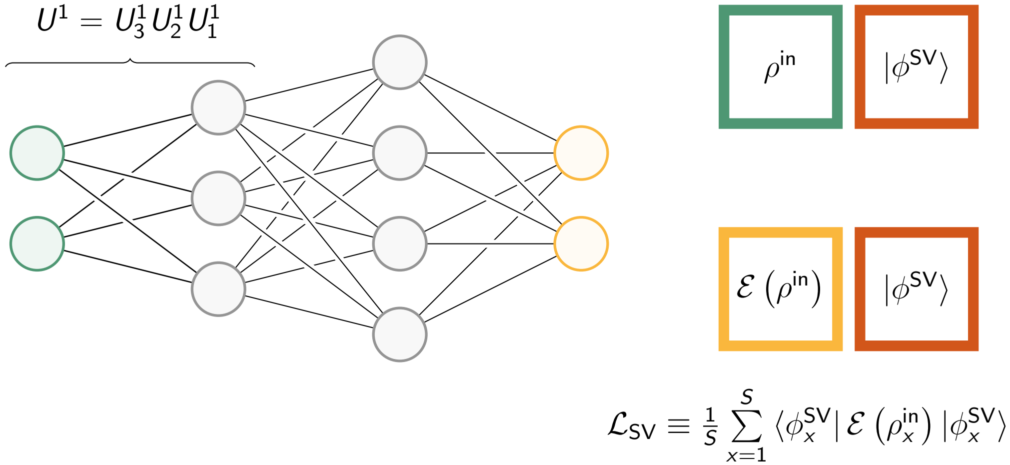
$$U_3^{1\dagger}\left(\mathbb{1}_{1,2}\otimes \mathrm{tr}_{\mathrm{out}}\left(U_1^{3\dagger} U_2^{3\dagger}\left(\mathbb{1}_{3,4,5}\otimes |\phi_x^{\mathrm{SV}}\rangle\langle\phi_x^{\mathrm{SV}}|\right) U_2^3 U_1^3\right)\right) U_3^1\Big]$$



$$K_2^1 = \frac{2^2 \eta}{S} \sum_{x=1} \mathrm{tr}_{\mathrm{rest}} \Big[U_2^1 U_1^1 \left(\rho_x^{\mathrm{in}} \otimes |000\rangle \langle 000| \right) U_1^{1\dagger} U_2^{1\dagger},$$

$$U_3^{1\dagger} \left(\mathbb{1}_{1,2} \otimes \mathrm{tr}_{\mathrm{out}} \left(U_1^{3\dagger} U_2^{3\dagger} \left(\mathbb{1}_{3,4,5} \otimes |\phi_x^{\mathrm{SV}}\rangle \langle \phi_x^{\mathrm{SV}}| \right) U_2^3 U_1^3 \right) \right) U_3^1 \Big]$$




$U^1 = U_3^1 U_2^1 U_1^1$
ρ^{in}
$|\phi^{\text{SV}}\rangle$
$\mathcal{E}(\rho^{\text{in}})$
$|\phi^{\text{SV}}\rangle$
$\mathcal{L}_{\text{SV}} \equiv \frac{1}{S}\sum_{x=1}^{S} \langle\phi_x^{\text{SV}}|\,\mathcal{E}(\rho_x^{\text{in}})\,|\phi_x^{\text{SV}}\rangle$

$$\Big\{ \left(\rho_1^{\text{in}}, |\phi_1^{\text{SV}}\rangle\langle\phi_1^{\text{SV}}|\right), \ldots, \left(\rho_S^{\text{in}}, |\phi_S^{\text{SV}}\rangle\langle\phi_S^{\text{SV}}|\right),$$

$$\left(\rho_{S+1}^{\text{in}}, |\phi_{S+1}^{\text{USV}}\rangle\langle\phi_{S+1}^{\text{USV}}|\right), \ldots, \left(\rho_N^{\text{in}}, |\phi_N^{\text{USV}}\rangle\langle\phi_N^{\text{USV}}|\right) \Big\}$$

$$\text{training loss } \mathcal{L}_{\mathrm{SV}} = \frac{1}{S}\sum_{x=1}^{S} \langle \phi_x^{\mathrm{SV}} | \, \mathcal{E}\left(\rho_x^{\mathrm{in}}\right) | \phi_x^{\mathrm{SV}} \rangle$$

$$\Big\{ \left(\rho_1^{\mathrm{in}}, |\phi_1^{\mathrm{SV}}\rangle \langle \phi_1^{\mathrm{SV}}|\right), \ldots, \left(\rho_S^{\mathrm{in}}, |\phi_S^{\mathrm{SV}}\rangle \langle \phi_S^{\mathrm{SV}}|\right),$$
$$\left(\rho_{S+1}^{\mathrm{in}}, |\phi_{S+1}^{\mathrm{USV}}\rangle \langle \phi_{S+1}^{\mathrm{USV}}|\right), \ldots, \left(\rho_N^{\mathrm{in}}, |\phi_N^{\mathrm{USV}}\rangle \langle \phi_N^{\mathrm{USV}}|\right) \Big\}$$

$$\text{training loss } \mathcal{L}_{\mathsf{SV}} = \frac{1}{S}\sum_{x=1}^{S} \langle \phi_x^{\mathsf{SV}} | \, \mathcal{E}\left(\rho_x^{\mathsf{in}}\right) | \phi_x^{\mathsf{SV}} \rangle$$

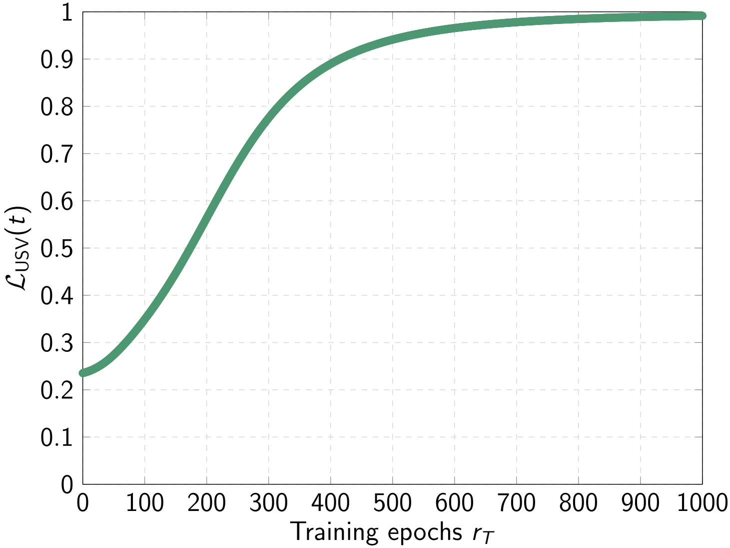
$$\text{validation loss } \mathcal{L}_{\mathsf{USV}} = \frac{1}{N-S}\sum_{x=S+1}^{N} \langle \phi_x^{\mathsf{USV}} | \, \mathcal{E}\left(\rho_x^{\mathsf{in}}\right) | \phi_x^{\mathsf{USV}} \rangle$$

$$\Big\{ \left(\rho_1^{\mathsf{in}}, |\phi_1^{\mathsf{SV}}\rangle\langle\phi_1^{\mathsf{SV}}|\right), \ldots, \left(\rho_S^{\mathsf{in}}, |\phi_S^{\mathsf{SV}}\rangle\langle\phi_S^{\mathsf{SV}}|\right),$$
$$\left(\rho_{S+1}^{\mathsf{in}}, |\phi_{S+1}^{\mathsf{USV}}\rangle\langle\phi_{S+1}^{\mathsf{USV}}|\right), \ldots, \left(\rho_N^{\mathsf{in}}, |\phi_N^{\mathsf{USV}}\rangle\langle\phi_N^{\mathsf{USV}}|\right) \Big\}$$

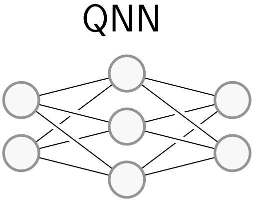
training data pairs

$$|\phi^{\mathsf{in}}\rangle$$

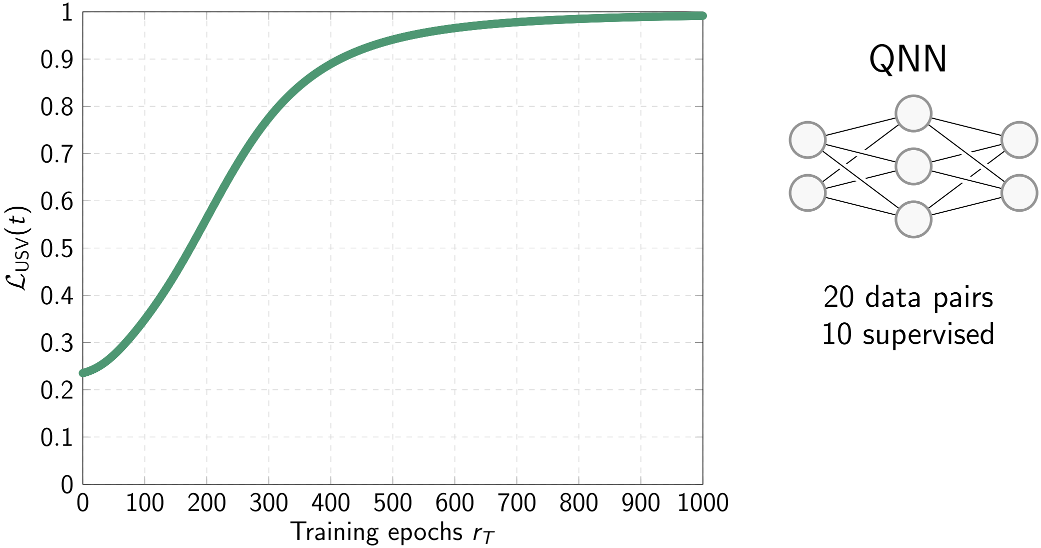
$$|\phi^{\mathsf{SV}}\rangle = Y\,|\phi^{\mathsf{in}}\rangle$$



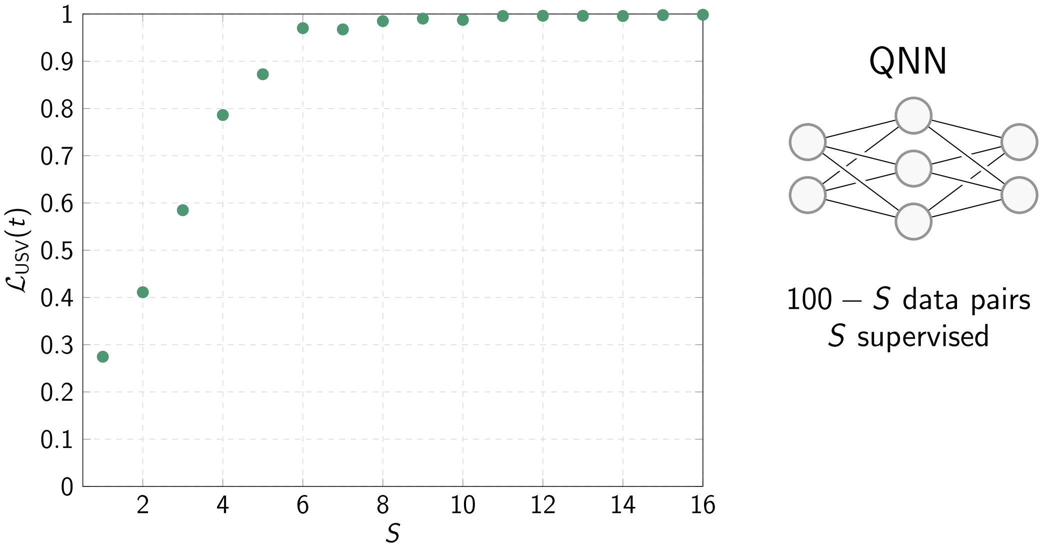
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$\mathcal{L}_{\mathsf{USV}}(t)$
0
100
200
300
400
500
600
700
800
900
1000
Training epochs r_T



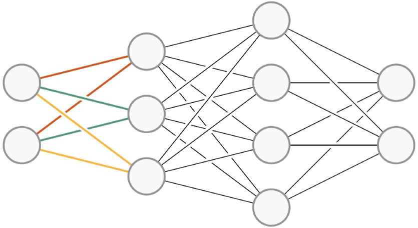
QNN

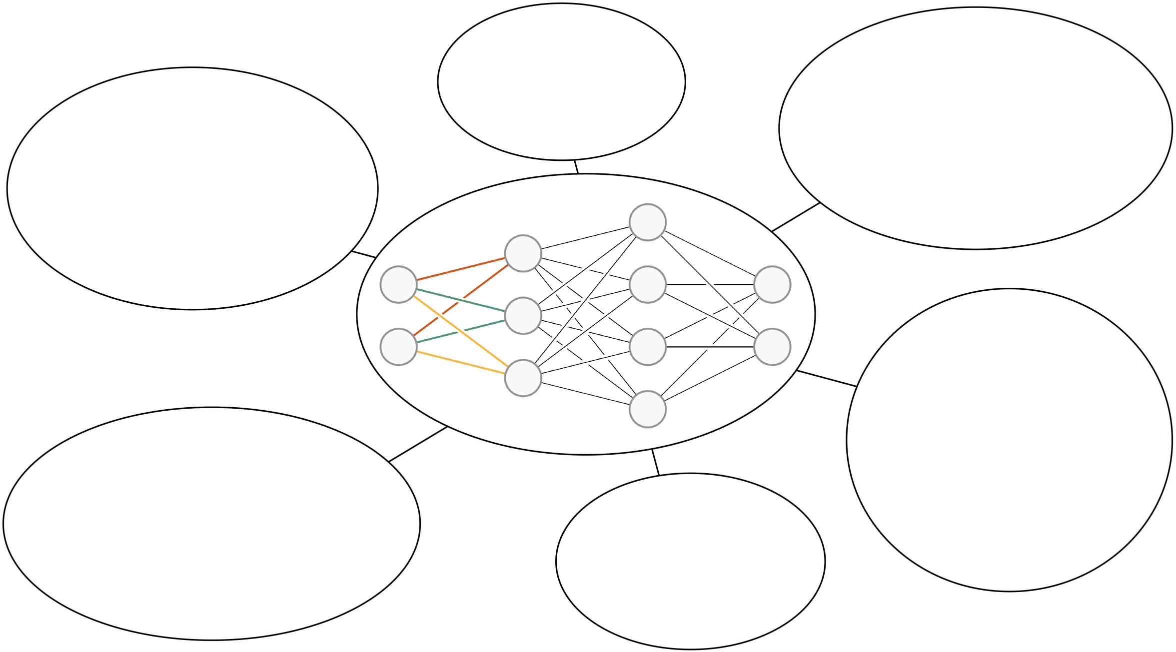


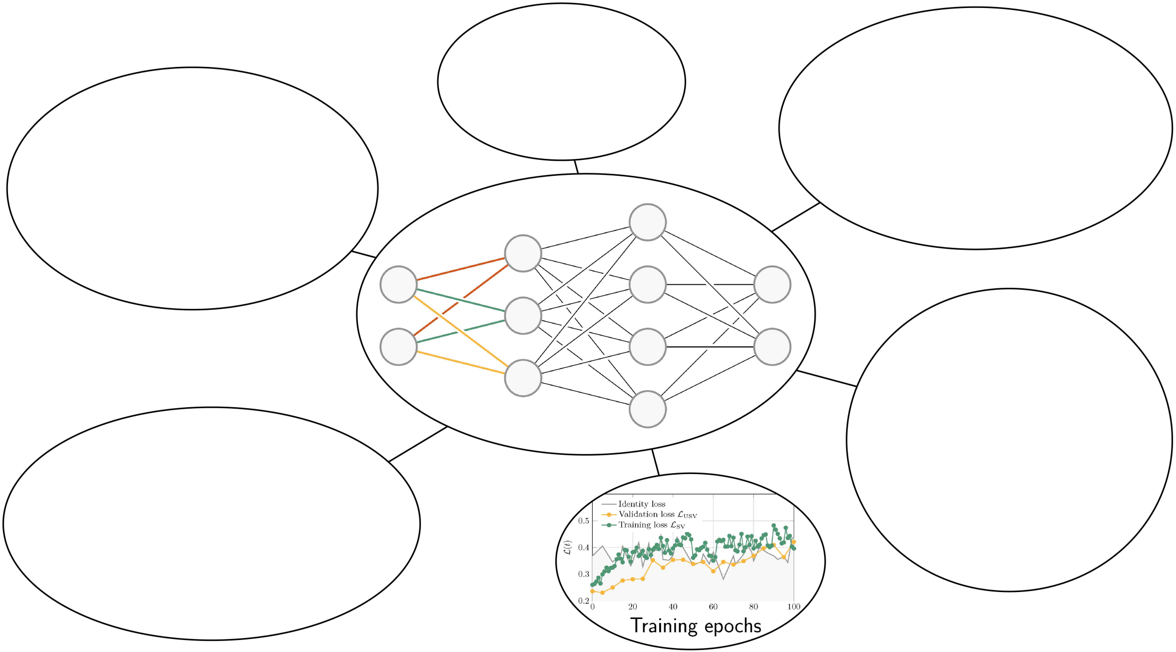
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
100
200
300
400
500
600
700
800
900
1000
$\mathcal{L}_{\mathrm{USV}}(t)$
Training epochs r_T
QNN
20 data pairs
10 supervised



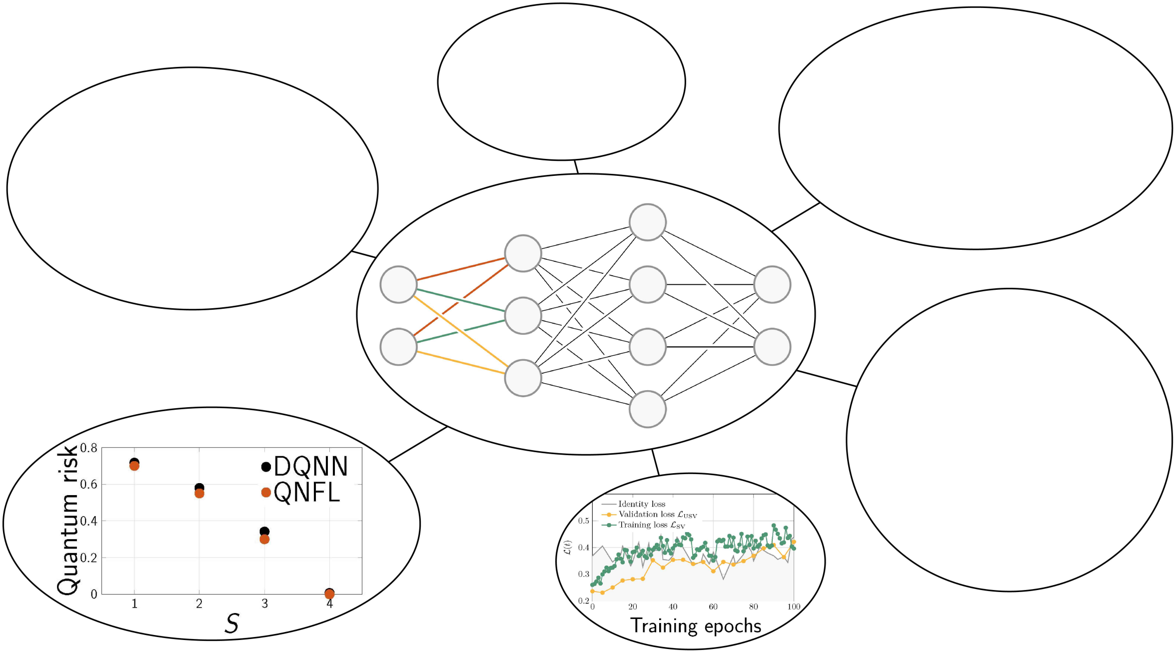
$\mathcal{L}_{\mathrm{USV}}(t)$
S
QNN
$100 - S$ data pairs
S supervised



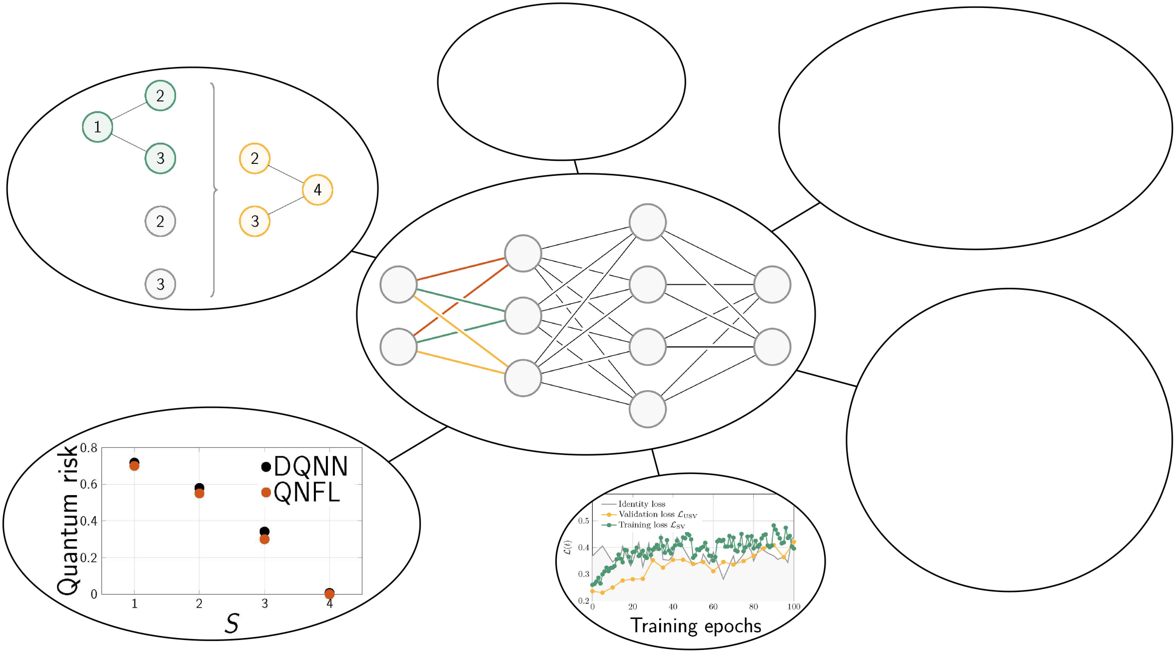




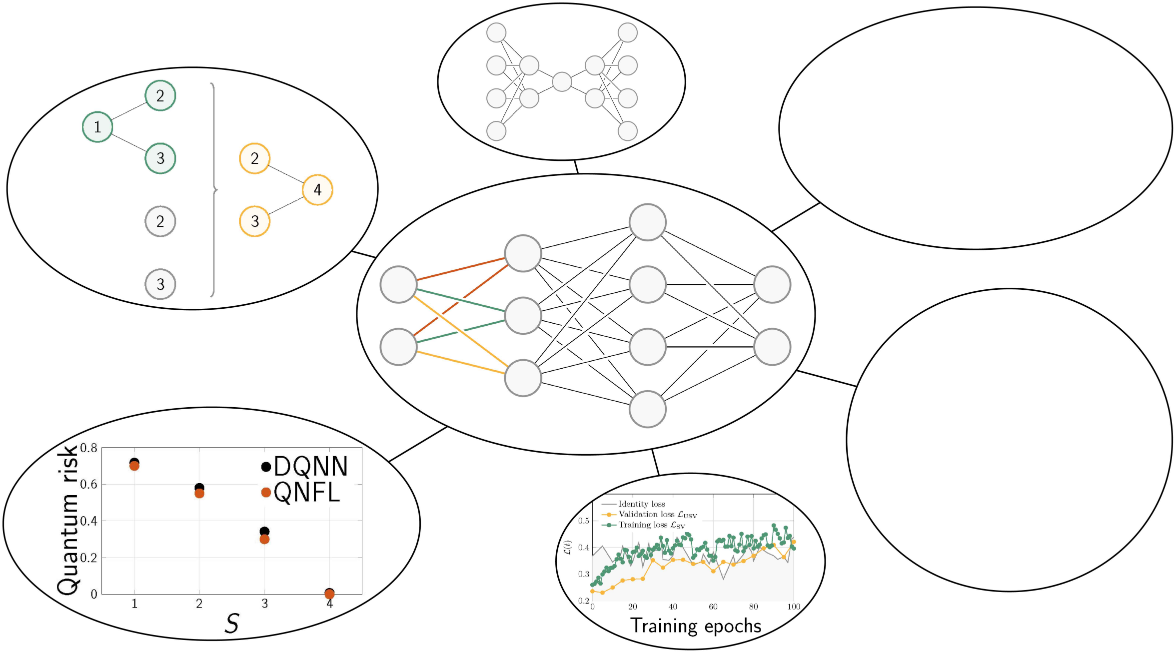
Identity loss
Validation loss $\mathcal{L}_{USV}$
Training loss $\mathcal{L}_{SV}$
$\mathcal{L}(t)$
0.2
0.3
0.4
0.5
0
20
40
60
80
100
Training epochs



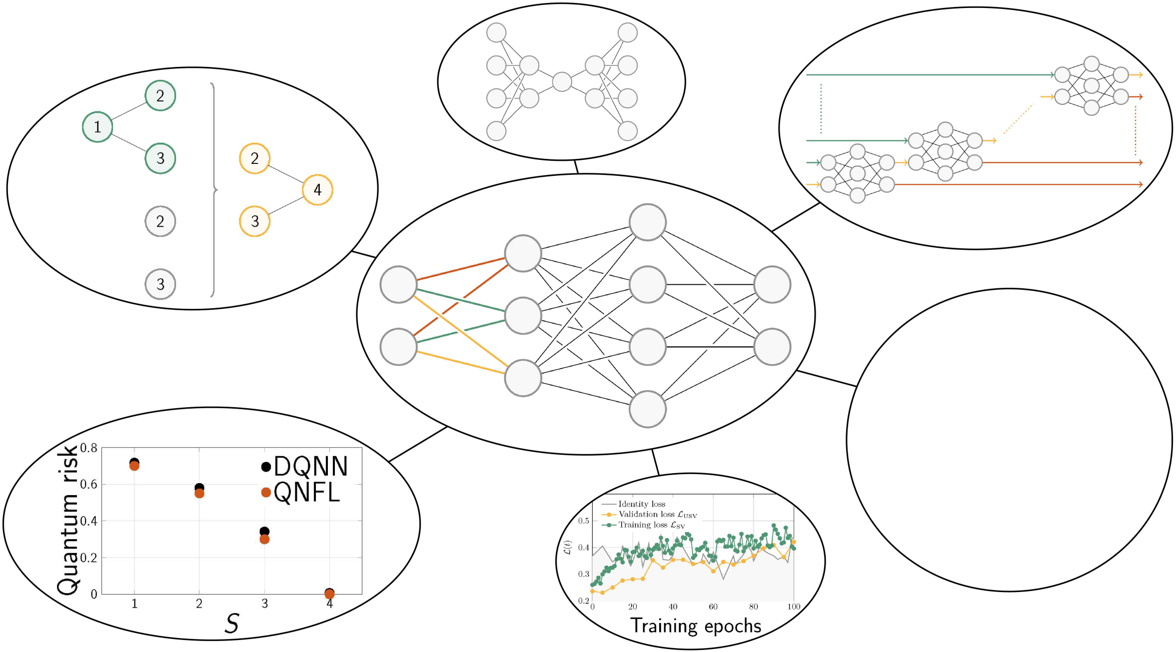
Quantum risk
DQNN
QNFL
S
Identity loss
Validation loss $\mathcal{L}_{USV}$
Training loss $\mathcal{L}_{SV}$
$\mathcal{L}(t)$
Training epochs



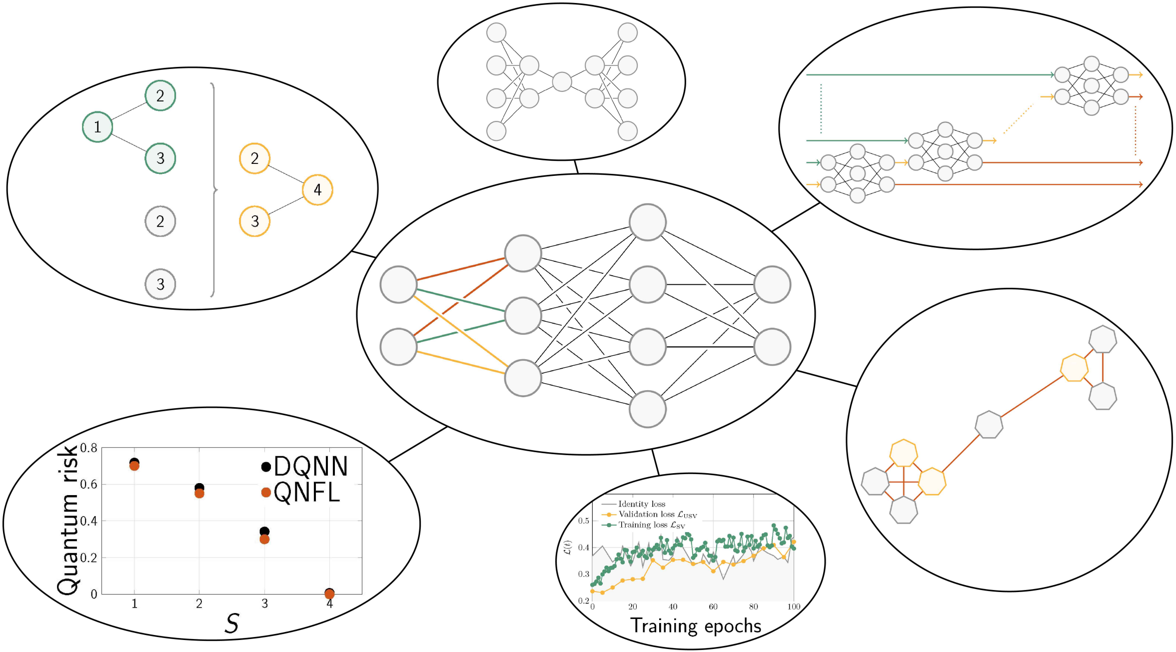
1
2
3
2
3
2
3
4
Quantum risk
0.8
0.6
0.4
0.2
0
1
2
3
4
S
DQNN
QNFL
Identity loss
Validation loss $\mathcal{L}_{\mathrm{USV}}$
Training loss $\mathcal{L}_{\mathrm{SV}}$
$\mathcal{L}(t)$
0.5
0.4
0.3
0.2
0
20
40
60
80
100
Training epochs



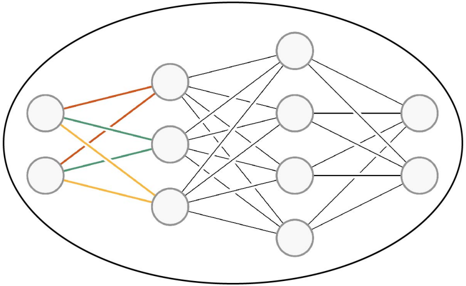
1
2
3
2
3
2
4
3
Quantum risk
0.8
0.6
0.4
0.2
0
1
2
3
4
S
DQNN
QNFL
Identity loss
Validation loss $\mathcal{L}_{\mathrm{USV}}$
Training loss $\mathcal{L}_{\mathrm{SV}}$
$\mathcal{L}(t)$
0.5
0.4
0.3
0.2
0
20
40
60
80
100
Training epochs

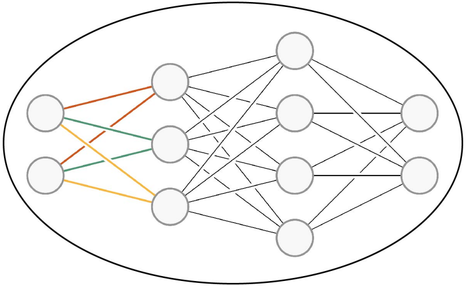


1
2
3
2
3
2
3
4
Quantum risk
DQNN
QNFL
S
Identity loss
Validation loss $\mathcal{L}_{\text{USV}}$
Training loss $\mathcal{L}_{\text{SV}}$
$\mathcal{L}(t)$
Training epochs



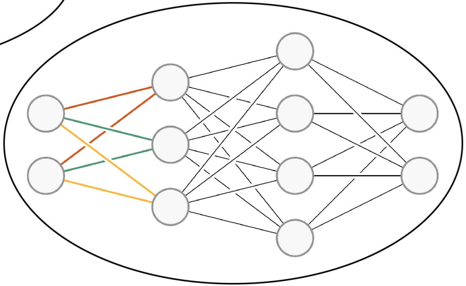
1
2
3
2
3
2
3
4
0.8
0.6
0.4
0.2
0
Quantum risk
1
2
3
4
S
DQNN
QNFL
Identity loss
Validation loss $\mathcal{L}_{\text{USV}}$
Training loss $\mathcal{L}_{\text{SV}}$
0.5
0.4
0.3
0.2
$\mathcal{L}(t)$
0
20
40
60
80
100
Training epochs



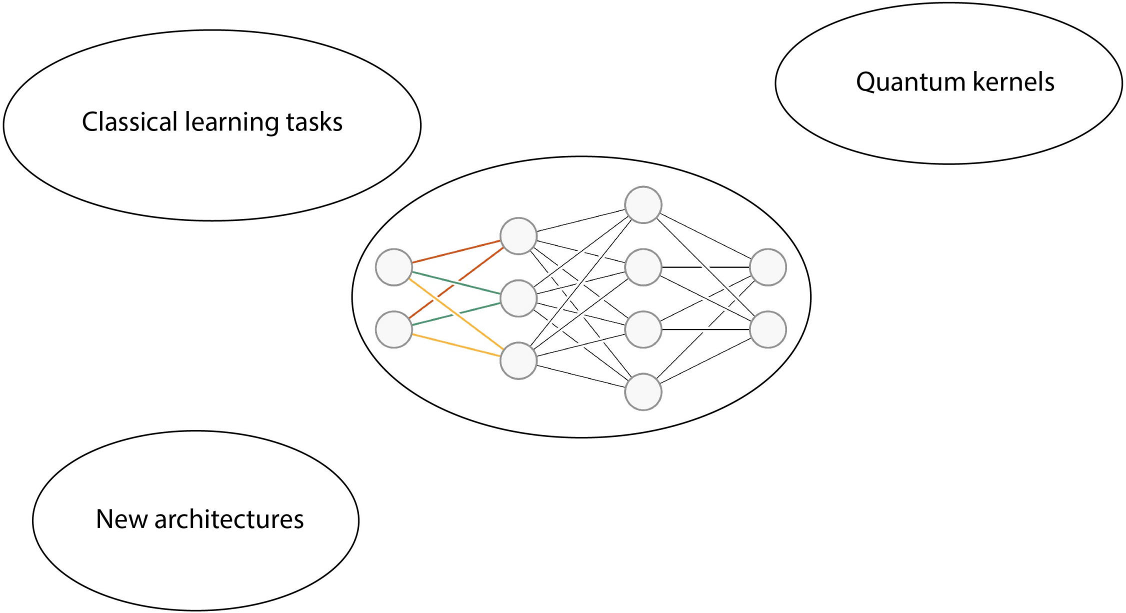


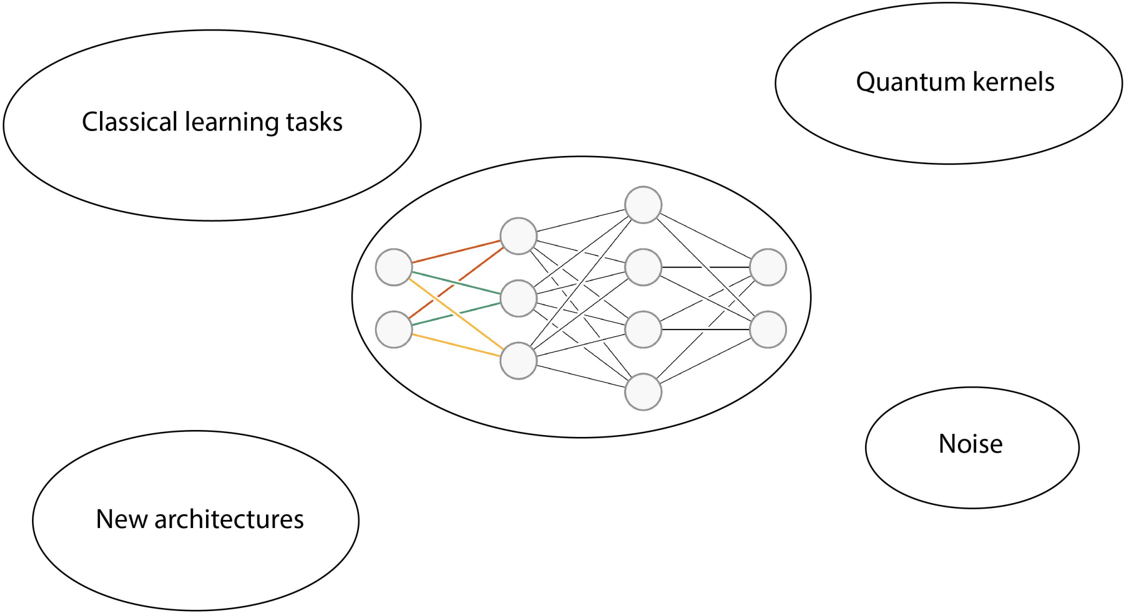
New architectures

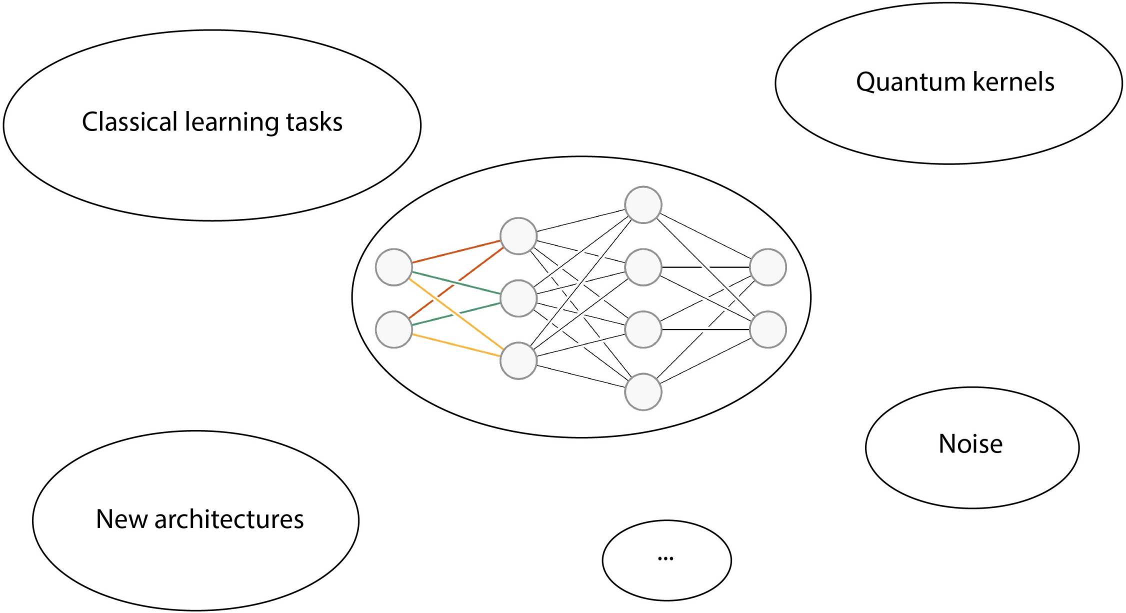
Classical learning tasks



New architectures


Classical learning tasks
Quantum kernels
New architectures


Classical learning tasks
Quantum kernels
Noise
New architectures



Classical learning tasks
Quantum kernels
Noise
New architectures
...